Dissipative Observer Design for bioprocesses

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Abstract. Recently, the author has proposed a methodology for the design of nonlinear observers based on the dissipative theory. This methodology offers a systematic approach to the observer design providing great flexibility and generality. For example, several well known observer design methods, as the High-Gain and the Lipschitz Observers, can be treated and generalized in a unified manner by the Dissipative Approach. Moreover, different objectives in observation can be also unified and generalized by the Dissipative Approach, as for example the design of Unknown Input and Robust Observers. The objective of this paper is to show how this methodology can be applied in the design of observers for bioprocesses and its advantages for this kind of processes. An example illustrates the main ideas.

1 Introduction

Reaction systems is a class of nonlinear dynamical systems that is widely used in areas such as chemical, biochemical and biomedical engineering, biotechnology, ecology, etc. (Robust) observation issues for this class of systems is of fundamental importance due to the limited availability of on-line sensors and the uncertainties related, in particular, to the mathematical model. It is not surprising that there is an intensive research activity to design observers (or software sensors) for these systems ([1–3]), and different methods for uncertain reaction systems, besides the classical extended Kalman and Luenberger observers, have been proposed (for an overview see [2]): Interval Observers ([4]) are based on cooperative systems theory; Adaptive Observers ([3]) assume that the uncertainties are represented by unknown parameters; Asymptotic Observers ([1, 2]) are based on the mass and energy balances without requiring the process kinetics; Practical and Parallelotopic Observers ([5]) consider uncertainties as *unknown inputs (UI)* and converge practically (not exactly) to the true state for a restricted class of systems with bounded perturbations.

For reaction systems without uncertainties several methods have been applied to design observers, as the High-Gain method ([6]) and the Lipschitz Method ([7]). For uncertain reaction systems, when there are only parametric uncertainties, adaptive observers can be used. However, if stronger structural uncertainties are available the most successful method used to day are the asymptotic

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observers ([1, 2]). In the work ([8]) uncertainties are represented by *arbitrary unknown input signals* to the system, what represents a flexible way to characterize many kinds of uncertainties, and they are able to show that the asymptotic observers can be recovered and extended with their approach. A highly satisfactory result is to be able to explain, using observability/detectability arguments, why (classic) asymptotic observers converge and why their convergence rate is not assignable. Moreover, the robust observers proposed in that work can be used in more general situations and their convergence properties are completely derived from the robust observability/detectability properties of the model.

However, due to the basic linear structure of the uncertain systems considered in ([8]), it is not possible to consider more general situations. For example, if some reaction rates are known but others are uncertain, this leads to a nonlinear structure with unknown inputs, that cannot be treated with that approach. So a natural extension of that work is to use unknown input observer design methods for uncertain reaction systems, and this is part of the objective of this paper.

The use of systems with unknown inputs for the representation of the uncertain reaction system's family leads naturally to the study of observability and detectability concepts for this kind of systems, and the construction and existence conditions of *Unknown Input Observers (UIO)*. For linear time invariant (LTI) systems this is a very well established topic ([9]), and some advances in the design of UIOs for nonlinear systems have been obtained recently ([10, 11])

Recently, the author has proposed ([12, 13]) a method to design nonlinear observers using dissipative methods. One attractive feature of this Dissipative Design is, on the one side, that it includes and generalizes many current observer design methods, and on the other side, that it is possible to design observers with unknown inputs or known inputs in a unified framework. The aim of this work is to show how the Dissipative Design Method can be used to design observers for reaction systems with or without uncertainties in a unified way. This can be seen as a first step in a more general, and far-reaching objective: to develop a methodology to design robust observers for uncertain reaction systems, in which different kinds of uncertainties are available as unknown constant parameters, unknown (bounded) disturbances, unmodeled dynamics, deterministic perturbations characterized by an internal model, etc. We believe that the Dissipative Design Method is able to reach these requirements, and this is part of active research work.

2 Dissipative observer design

Motivated by the circle criterion design of nonlinear observers in [14] the author has proposed in [12, 13] a methodology for designing nonlinear observers for a class of nonlinear systems. This method will be briefly reviewed in this section.

2.1 Preliminaries

From the dissipativity theory ([15]) the following results are of relevance here. Consider the feedback interconnection

$$\dot{x} = Ax + Bu \ , \ y = Cx \ , \ u = -\psi(t, y) \ , \tag{1}$$

with $x \in \mathbb{R}^n$, $u \in \mathbb{R}^q$, $y \in \mathbb{R}^m$, and quadratic *supply rates* $\omega(v, w) = v^T Q v + 2v^T S w + w^T R w$, where $v \in \mathbb{R}^r$, $w \in \mathbb{R}^s$, $Q \in \mathbb{R}^{r \times r}$, $S \in \mathbb{R}^{r \times s}$, $R \in \mathbb{R}^{s \times s}$, and Q, R symmetric. The linear part (A, B, C) of system (1) is said to be *state strictly dissipative (SSD)* with respect to the supply rate $\omega(y, u)$, or for short (Q, S, R)-SSD, if there exist a matrix $P = P^T > 0$, and $\epsilon > 0$ such that

$$\begin{bmatrix} PA + A^T P + \epsilon P & PB \\ B^T P & 0 \end{bmatrix} - \begin{bmatrix} C^T Q C & C^T S \\ S^T C & R \end{bmatrix} \leq 0 \, . \tag{2}$$

For quadratic systems, i.e. $m = q$, passivity corresponds to the supply rate $\omega(y, u) = y^T u$. Note that this definition assures the existence of a quadratic positive definite *storage function* $V(x) = x^T P x$ such that along any trajectory of the system $\dot{V}(x(t)) \leq \omega(y(t), u(t)) - \epsilon V(x(t))$.

The nonlinear part of system (1), a time-varying memoryless nonlinearity $\psi : [0, \infty) \times \mathbb{R}^m \rightarrow \mathbb{R}^q$, $u = \psi(t, y)$, piecewise continuous in t and locally Lipschitz in y, such that $\psi(t, 0) = 0$, is said to satisfy a dissipative condition in Γ with respect to the supply rate $\omega(u, y)$, or for short (Q, S, R)-D in Γ, if $\omega(u, y) = \omega(\psi(t, y), y) \geq 0$, $\forall t \geq 0$, $\forall y \in \Gamma \subseteq \mathbb{R}^m$, where Γ is a subset of $\mathbb{R}^m$ whose interior is connected and contains the origin. If $\Gamma = \mathbb{R}^m$, then ψ satisfies the dissipativity condition globally, in which case it is said that ψ is dissipative with respect to ω, or for short, (Q, S, R)-D.

Note that the classical sector conditions ([16]) for square nonlinearities, i.e. $m = q$, can be represented in this form. If ψ is in the sector $[K_1, K_2]$, i.e. $(y - K_1 u)^T (K_2 u - y) \geq 0$, then it is (Q, S, R)-D, with $(Q, S, R) = \left(-I, \frac{1}{2}(K_1 + K_2), -\frac{1}{2}\left(K_1^T K_2 + K_2^T K_1\right)\right)$. If ψ is in the sector $[K_1, \infty]$, i.e. $(y - K_1 u)^T u \geq 0$, then it is $\left(0, \frac{1}{2} I, -\frac{1}{2}\left(K_1 + K_1^T\right)\right)$-D.

For the interconnected system (1) a generalization of the passivity and of the small gain theorems for non square systems can be easily obtained, and it will be used in the sequel.

Lemma 1. *Consider the system (1). If the linear system (C, A, B) is $\left(-R, S^T, -Q\right)$-SSD, then the equilibrium point $x = 0$ of (1) is globally (locally) exponentially stable for every (Q, S, R)-D (in Γ for some $\Gamma \subseteq \mathbb{R}^m$) nonlinearity.*

2.2 Dissipative design for certain nonlinear systems

Consider the class of systems described by a LTI subsystem with a nonlinear perturbation term, connected in feedback, i.e.

$$\Sigma : \left\{ \dot{x} = Ax + G\psi(\sigma, y, u) + \gamma(t, y, u) \ , \ y = Cx \ , \ \sigma = Hx \ , \right. \tag{3}$$

or that can be brought to this form by transformations, and where $x \in \mathbb{R}^n$ is the state, $y \in \mathbb{R}^p$ is the measured output, $u \in \mathbb{R}^m$ is the input, and $\sigma \in \mathbb{R}^r$ is a (not necessarily measured) linear function of the state. $\gamma(t, y, u)$ is an arbitrary nonlinear function of the input and the output. $\psi(\sigma, y, u)$ is a q-dimensional vector that depends on σ, y, u. ψ and γ are assumed to be locally Lipschitz in σ, y, u, so that existence and uniqueness of solutions is guaranteed. It will be assumed that the trajectories of interest of Σ are defined for all future times.

An *observer* for system (3) is a dynamical system Ω that has as inputs the input u and the output y of Σ, and its output $\hat{x}$ is an estimation of the state x of Σ. A full order observer for Σ of the form

$$\Omega : \begin{cases} \dot{\hat{x}} = A\hat{x} + G\psi(\hat{\sigma} + N(\hat{y} - y), y, u) + L(\hat{y} - y) + \gamma(t, y, u), \\ \hat{y} = C\hat{x}\ , \hat{\sigma} = H\hat{x}\ , \end{cases} \tag{4}$$

is proposed, where matrices $L \in \mathbb{R}^{n \times p}$, and $N \in \mathbb{R}^{r \times p}$ have to be designed. Defining the state estimation error by $\tilde{x} \triangleq \hat{x} - x$, the output estimation error by $\tilde{y} \triangleq \hat{y} - y$, and the function estimation error by $\tilde{\sigma} \triangleq \hat{\sigma} - \sigma$, $z \triangleq (H + NC)\tilde{x} = \tilde{\sigma} + N\tilde{y}$, and a new nonlinearity$\phi(z, \sigma, y, u) \triangleq \psi(\sigma, y, u) - \psi(\sigma + z, y, u)$, the dynamics of the error can be written as

$$\Xi : \left\{ \dot{\tilde{x}} = A_L\tilde{x} + G\nu\ , \quad \tilde{x}(0) = \tilde{x}_0\ , z = H_N\tilde{x}\ , \nu = -\phi(z, \sigma, y, u)\ , \right. \tag{5}$$

where $A_L \triangleq A + LC$, and $H_N \triangleq H + NC$. $\phi(0, \sigma; y, u) = 0$ for all σ, y, u.

The observer design consists in finding matrices L and N, if they exist, so that Ξ satisfies the conditions of Lemma 1. For this it is necessary to assume that the nonlinear part of (5) belongs to one or several sectors.

Assumption 1 *ϕ in (5) is (Q_i, S_i, R_i)-dissipative (in Γ) for some finite set of non positive semidefinite quadratic forms $\omega_i(\phi, z) = \phi^T Q_i \phi + 2\phi^T S_i z + z^T R_i z \geq 0$, for all σ, y, u, for $i = 1, 2, \cdots, M$.*

It is clear that it is necessary that the quadratic forms be independent. It is also easy to see that then ϕ is $\sum_{i=1}^{M} \theta_i(Q_i, S_i, R_i)$-dissipative (in Γ) for every $\theta_i \geq 0$, i.e. ϕ is dissipative with respect to the supply rate $\omega(\phi, z) = \sum_{i=1}^{M} \theta_i \omega_i(\phi, z)$. In this case the design is as follows

Theorem 2. *Suppose that Assumption 1 is satisfied. If there are matrices L and N, and a vector $\theta = (\theta_1, \cdots, \theta_M)$, $\theta_i \geq 0$, such that the linear subsystem of Ξ is $(-R_\theta, S_\theta^T, -Q_\theta)$-SSD, with $(Q_\theta, S_\theta, R_\theta) = \sum_{i=1}^{M} \theta_i(Q_i, S_i, R_i)$, that is there exist a matrix $P = P^T > 0$, and $\epsilon > 0$ such that*

$$\begin{bmatrix} PA_L + A_L^T P + \epsilon P + H_N^T R_\theta H_N\ , & PG - H_N^T S_\theta^T \\ G^T P - S_\theta H_N & Q_\theta \end{bmatrix} \leq 0\ , \tag{6}$$

where $A_L = A + LC$, $H_N = H + NC$. Then Ω is a global (local) exponential observer for Σ, i.e. there exist constants $\kappa, \mu > 0$ such that for all $\tilde{x}(0)$ (in a vicinity of $\tilde{x} = 0$) $\|\tilde{x}(t)\| \leq \kappa \|\tilde{x}(0)\| \exp(-\mu t)$.

The proposed method generalizes and improves several methods previously proposed in the literature ([12]): (i) The Circle criterion design ([14]): our design is valid for non-square systems, the nonlinearities are of general type, and can be described by several sector conditions. (ii) Lipschitz observer design [7], and (iii) High-Gain observer design [6].

2.3 Dissipative design for uncertain nonlinear systems

One alternative to model uncertain systems consists in considering the uncertainties as completely unknown inputs to the system. The class of nonlinear systems considered for UIO design is

$$\Sigma : \left\{ \dot{x} = Ax + G\psi(\sigma, y, u) + \gamma(t, y, u) + Bw \ , \ y = Cx \ , \ \sigma = Hx \ , \right. \tag{7}$$

where $w \in \mathbb{R}^q$ is an arbitrary (even unbounded) unknown input. w can model an arbitrary unknown disturbance acting on the system, parametric uncertainty or unmodeled dynamics. It will be assumed that the trajectories of Σ exist and are well defined for all times, i.e. there are no finite escape times. Without loss of generality it is assumed that matrices B and C are of full rank.The objective is to design an *Unknown Input Observer (UIO)* for system Σ (7), that is, a dynamical system that using the information of the known input $u(t)$ and the output $y(t)$ produces an state estimate $\hat{x}(t)$, that converges asymptotically to the actual state $x(t)$ of Σ, i.e. $\lim_{t\to\infty}(\hat{x}(t) - x(t)) = 0$, in spite of the lack of information on the unknown input w and derivative(s) of output y.

The main result of [17, 10] is a sufficient condition for the existence of an UIO for the plant Σ (7).

Theorem 3. *Suppose that Assumption 1 is satisfied, and that there exist constant matrices $P = P^T > 0$, L, N, $\mathbb{S}$, a vector $\theta = (\theta_1, \cdots, \theta_M)$, $\theta_i \geq 0$, and a constant $\epsilon > 0$, such that (★ represent the symmetric terms)*

$$\begin{bmatrix} PA_L + A_L^T P + \epsilon P + H_N^T R_\theta H_N & \bigstar & \bigstar \\ G^T P - S_\theta H_N & Q_\theta & \bigstar \\ B^T P - \mathbb{S}C & 0 & 0 \end{bmatrix} \leq 0 \, .$$

Then there exists an UIO for (7).

As it is shown in the references if this conditions are satisfied there are state $\chi = Tx$ and output transformations such that in the new coordinates the system has the form

$$\dot{\chi}_1 = \bar{A}_{11}\chi_1 + \bar{A}_{12}\chi_2 + \bar{G}_1\psi(\sigma, y, u) + \bar{\gamma}_1 - B^T PBw \tag{8}$$

$$\dot{\chi}_2 = \bar{A}_{21}\chi_1 + \bar{A}_{22}\chi_2 + \bar{G}_2\psi(\sigma, y, u) + \bar{\gamma}_2 \tag{9}$$

$$y_1 = \chi_1 \ , \ y_2 = C_2\chi_2 \ , \ \sigma = H_2\chi_2 \, .$$

Note that the states affected by the unknown input (χ_1) (8) are measured, and the estimation of the rest of the states (9), that are unaffected by the unknown input (χ_2), can be performed as in the previous subsection.

Remark 1. In general (6, 3) are nonlinear matrix inequality feasibility problems. Under some conditions they become Linear Matrix Inequalities (LMI) feasibility problems, for which solutions can be effectively found by several algorithms in the literature ([18]). Note also that when (6, 3) are feasible, there exist in general several solutions for L and N. Replacing ϵP by ϵI, (3) is a LMI in P, PL, ϵ, $\mathbf{S}$, θ but not in N, except when $R_\theta = 0$ and $S_\theta = 0$. One possibility to solve (6, 3) by LMI algorithms is to fix N at some value and to search for a solution. This can be made recursively until a solution is found. A particular situation arises when $N = 0$, so that the classical output injection is made.

3 Model of (Uncertain) Reaction Systems and robust observer design

A general state-space model of reaction systems is generally obtained from mass and energy balances ([1, 2]) and can be written in a compact and generalized form as:

$$\Sigma_R : \left\{ \dot{x} = K\varphi(x) - D(t)x - Q(x) + F(t) \right. , y = Cx . \tag{10}$$

where $y \in \mathbb{R}^m$ is the output vector, the state $x \in \mathbb{R}^n$ consists of component concentrations, volumes and temperatures, $K \in \mathbb{R}^{n\times q}$ is the constant stoichiometric coefficient matrix, $\varphi \in \mathbb{R}^q$ is the reaction rate vector, D is the (matrix) dilution rate, Q is the outflow rate vector, F is the feedrate vector. For a single reactor D is a scalar but it is a matrix when several reactors are considered.

In practice the model is usually uncertain, since the parameters and nonlinearities of the system are difficult to identify precisely and they may change over time. In particular, the reaction rates are usually poorly known. This makes the observation problem challenging. In order to deal with these uncertainties a representation of all possible behaviors of the system (10) is required. In a previous work ([8]) the authors have proposed to use state-affine systems with *unknown inputs*, that is

$$\Sigma_U : \left\{ \dot{x} = A(u,y)x + Bw + \psi(u,y) \right. , \ y = Cx , \tag{11}$$

where $w \in \mathbb{R}^p$ is a vector of (arbitrary) unknown inputs representing uncertainties, $u \in \mathbb{R}^r$ is a vector of measured inputs, and $A(u,y)$ is a continuous matrix. In this form they have been able to explain and generalize the well known asymptotic observers, that have been shown to be very useful in many practical situations ([1, 19, 2]). They are obtained when all the reaction rates are considered uncertain, $w = \varphi(x)$, but the rest of the model is assumed to be known, i.e. the uncertain system can be represented by (11) with $A(u,y) = -D(t)$, $B = K$, $\psi(u,y) = F(t) - Q(x)$.

However, due to the basic linear structure of (11) it is not possible to consider more general situations. For example, if some reaction rates are known but others are uncertain, this leads to a nonlinear structure as the one in (7), when the uncertain reaction rates are modeled as unknown inputs. It seems also natural to use (7) as a model for an uncertain reaction system. The dissipative method can then be used to design a robust observer. It is clear that the classic asymptotic observers are a special case of this approach. Moreover, the case that no uncertainties are present in the model can be treated in the same framework. This shows the great flexibility of the method.

Example 1. Consider the case that in system (10) some reaction rates ($\varphi_k(x)$) are well known but the rest is unknown ($\varphi_u(x)$). If it is assumed that Q, D and F are measured, and K is known, then the reaction system can be written as

$$\dot{x} = K_k \varphi_k(x) - D(t)\,x - Q(x) + K_u w + F(t) \ , \ y = Cx \ .$$

in which $w = \varphi_u(x)$. This system has the structure of (7).

4 Example

In order to illustrate the dissipative observer design method proposed a simple biological reactor model will be considered:

$$\dot{X} = -D(t)\,X + \mu(S)\,X \ , \ \dot{S} = D(t)\,(S_{in} - S) - \tfrac{1}{Y}\mu(S)\,X \ , \qquad (12)$$

where X is the biomass and S the substrate concentration, μ is the growth rate, Y the yield coefficient, S_{in} is the substrate concentration in the inflow and D is the dilution rate. The observation problem consists in estimating the substrate concentration S when the biomass concentration X is measured. Two extreme conditions on the knowledge of the reaction rate will be considered:
Case 1: The reaction rate μ is completely unknown.
Case 2: No uncertainty, i.e. the model is perfectly known.

4.1 Case 1: Unknown reaction rate

Case 1 is the standard situation for asymptotic observers, where μ is treated as an unknown input ([1, 2, 8]). Since there is only one reaction rate and one measurement in this example, the dissipative approach leads exactly to the classical asymptotic observer. The variable $Z = \frac{1}{Y}X + S$, whose dynamics is$\dot{Z} = -D(t)\,Z + D(t)\,S_{in}$, is independent of the reaction rate μ. The asymptotic observer

$$\dot{\hat{Z}} = -D(t)\,\hat{Z} + D(t)\,S_{in} \ ,$$

converges asymptotically to the true value of Z, independently of the value of μ, when D is persistently exciting, i.e. there exist $\alpha, T > 0$ such that for all $t \geq 0$, $\int_t^{t+T} D(\tau)d\tau \geq \alpha$. The convergence of the observer cannot be assigned and depends on the behavior of D. The detectability analysis of [8] shows that if μ is completely arbitrary no better result can be obtained.

4.2 Case 2: Known reaction rate

Consider the following observer

$$\begin{aligned} \dot{\hat{X}} &= -D(t)X + \mu\left(\hat{S} + N\left(\hat{X} - X\right)\right)X + l_1\left(\hat{X} - X\right) , \\ \dot{\hat{S}} &= D(t)\left(S_{in} - \hat{S}\right) - \tfrac{1}{Y}\mu\left(\hat{S} + N\left(\hat{X} - X\right)\right)X + l_2\left(\hat{X} - X\right) . \end{aligned} \tag{13}$$

The dynamics of the observation errors $e_X = \hat{X} - X$, $e_S = \hat{S} - S$ are

$$\begin{aligned} &\dot{e}_X = l_1 e_X + \phi(z,S)X \ , \ \dot{e}_S = -D(t)e_S - \tfrac{1}{Y}\phi(z,S)X + l_2 e_X \ , \\ &z = e_S + N e_X \ , \ \phi(z,S) = \mu(z+S) - \mu(S) \ . \end{aligned} \tag{14}$$

It is illustrative to use (e_X, z) as state variables of the error, instead of (e_X, e_S), i.e.

$$\begin{aligned} &\dot{e}_X = l_1 e_X + \phi(z,S)X \ , \\ &\dot{z} = -D(t)z + \left(N - \tfrac{1}{Y}\right)\phi(z,S)X + l_N e_X \ , l_N = N l_1 + l_2 \ . \end{aligned} \tag{15}$$

In order to design the observer the sector of ϕ has to be determined. For continuous differentiable reaction rates μ this is easily done with the help of the mean value theorem. Since $\phi(z,S) = \frac{d\mu(S+\gamma z)}{dS} z$ for some $\gamma \in (0,1)$ it follows that $\phi(z,S)$ is in the sector $[K_1, K_2]$, where K_1 and K_2 are the minimum and the maximum value of the derivative of μ, respectively.

Two classical classes of growth rates will be considered: **The monotonic case:** The typical form is the Monod function $\mu(S) = \frac{\mu_0 S}{S+K_S}$, but other forms are possible. In this case $0 < K_1 < K_2 < \infty$. The strict positiveness of K_1 comes from the fact that in the reactor S is bounded.

The non monotonic case: The typical form is the Haldane function $\mu(S) = \frac{\mu_0 S}{S^2/K_i + S + K_S}$, but other forms are possible. In this case $K_1 < 0 < K_2 < \infty$.

It is possible to design the observer gains by solving the Matrix Inequality (6), that, in general, has many solutions, if it is feasible. Here, for illustrative purposes, a simple storage function will be selected and the design parameters will be selected to satisfy the corresponding inequality.

Consider as Lyapunov's function candidate $V(e_X, z) = \frac{1}{2}\left(e_X^2 + \theta z^2\right)$. Its derivative along the trajectories of the observation error 15 is

$$\dot{V} = l_1 e_X^2 + X\phi(z,S)e_X + \theta l_N z e_X - \theta D(t) z^2 + \theta X\left(N - \tfrac{1}{Y}\right)\phi(z,S)z \ .$$

The design is then as follows:

The monotonic case: Selecting $l_1 = \lambda_1 X$, $\theta > 0$, $l_N = \lambda_N X$ and $N < \frac{1}{Y}$, with λ_1 large enough, then $\dot{V} < -\epsilon V$ so that the observer converges exponentially fast, even when $D = 0$.

The non monotonic case: Selecting $l_1 = \lambda_1 X$, $\theta > 0$, $l_N = \lambda_N X$ and $N = \frac{1}{Y}$, with λ_1 large enough, then $\dot{V}$ is negative definite, so that the observer converges asymptotically fast, if it is assumed that $D(t) \geq \epsilon > 0$.

Many more solutions can be found solving the Matrix Inequality (6). These degrees of freedom can be used to optimize certain performance criteria. Note that setting $N = \frac{1}{Y}$ and $l_N = 0$ in the previous observer, then the asymptotic observer is recovered!

In Figure 1 some simulations illustrate the behavior of the designed observers. The growth rate is Monod and the parameters of the plant: $Y = 0.3$, $S_{in} = 10$, $\mu_0 = 0.2$, $K_S = 10$, $D = 0.4$, $X_0 = 10$, $S_0 = 5$. For both observers $\theta = 0.025$, $l_1 = -3X, e_{X0} = 0.5, e_{S0} = 10$ were used. For the asymptotic observer $N = 1/Y$ and $l_N = 0$, and for the dissipative observer $N = 1/Y - 8$, $l_N = -X/\theta$.

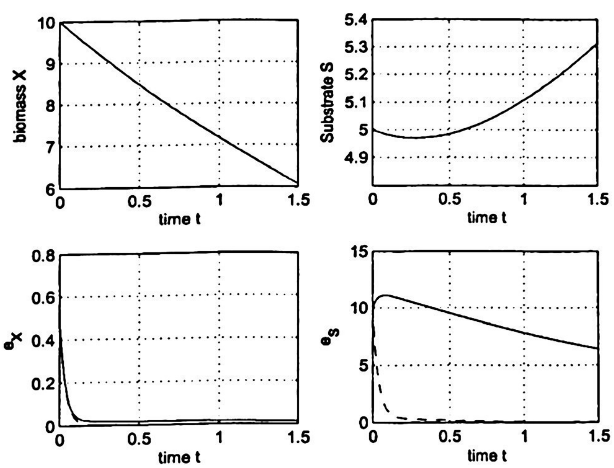

Fig. 1. Simulation of the bioreactor and the estimation errors of the asymptotic observer (continuous line) and of the dissipative observer (dotted line).

It is clear that the convergence velocity of the error of the unmeasured state (S) for the dissipative observer is much faster than that of the asymptotic observer. This is of course expected, since the model is perfectly known for the first but not for the second one. The interesting point here is that the dissipative observer methodology allows for a unified design under different uncertainty conditions.

5 Conclusions

In this work it has been shown how the Dissipative Design Method can be used to design observers for reaction systems with or without uncertainties in a unified way. Many important issues as the consideration of unknown parameters, sensor noise, consideration of trade offs between robustness and observer performance, etc. have to be addressed and this is part of active research work. We believe that the Dissipative Design Method is a methodology able to reach these requirements.

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